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Particle Creation in Anisotropically Expanding Universe

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Using squeezed vacuum states formalism of quantum optics, a homogeneous and massive scalar field minimally coupled to gravity in Bianchi type-I model of the universe is examined in the frame work of semiclassical theory of gravity. Hence an approximate leading solution to the semiclassical Einstein equation is found. The next order solution for each scale factor in their respective direction show power law of expansion. It is further noted that evolution of scale factors are mutually correlated. The phenomena of nonclassical particle creation is also examined in the anisotropic background cosmology.

KEY WORDS: Bianchi type-I; particle creation; requeezed states; cosmology.

1. INTRODUCTION

The present universe in its over all structure seems to be spatially homogeneous and isotropic but there are reasons to believe that it has not been so during its evolution and that inhomogeneities and anisotropies might have played an important role in the early universe (Misner, 1969; Misner *et al*., 1973). The isotropic model is adequate enough for the description of later stages of evolution of the universe but this does not mean that the model equally suite for the description of very early stages of evolution of the universe, especially near the singularity (Misner *et al*., 1973). Also the most general solution of the problem of gravitational collapse turn to be locally anisotropic near the singularity (Belinski *et al*., 1972; Heckman and Schucking, 1962; Throne, 1967). Cosmological solutions of Einstein's general relativity are also known in which the expansion can be anisotropic at first, near the singularity, and later the expansion became isotropic. Also, to avoid postulating specific initial conditions as well as the existence of particle horizon in isotropic models, attempts have been made through the study of inhomogeneous and anisotropic models of the universe. Among the anisotropic

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cosmological models, Bianchi type-I universe is the simplest one. In this model the metric is considered as spatially homogeneous and possibly anisotropic. In contrast to the Friedmann–Robertson–Walker (FRW) metric, Bianchi type-I metric has three scale factors which evolve differently in their respective direction. Therefore the expansion in this model can be considered as anisotropic expansion. Interests in such models have been received much attention (Belinski *et al*., 1971; Heckman and Schucking, 1962; Hu and Parker, 1978; Misner, 1969; Throne, 1967; Zel'dovich and Starobinsky, 1971). Huang has considered the fate of symmetry in Bianchi type-I cosmology using adiabatic approximation for massless field with arbitrary coupling to gravity (Huang, 1990). Futamase has studied the effective potential in Bianchi type-I cosmology (Futamas, 1984). Berkin has examined the effective potential in Bianchi type-I universe, for scalar field having arbitrary mass and coupling to gravity (Berkin, 1992). These studies show that Bianchi type-I cosmological model may be useful to understand the very early universe. Anisotropic models of the universe which become isotropic have been considered several times (Belinski and Khalatnikov, 1972). These motivate the study of anisotropic background cosmological models with scalar field possessing the advantage of FRW model (Folomeev and Gurovich, 2000). The possibility of the Bianchi type-I universe approaches to the isotropic model can be examined. From anisotropic to isotropic transition, a damping mechanism is required. One of the efficient damping mechanisms could be due to particle creation in anisotropic models. Therefore it would be useful to examine particle creation in anisotropic cosmological model with nonclassical inflaton, which could expect to produce sufficient particles to bring isotropy during the evolution process of the universe.

In this paper we study a homogeneous massive scalar field (inflaton), minimally coupled to gravity, in a spatially homogeneous and anisotropic background metric. The inflaton under our consideration can be quantized and represented in squeezed vacuum states, hence an approximate solution to the semiclassical Einstein equation and the phenomenon of nonclassical particle creation can be examined in semiclassical theory of gravity. Throughout the paper, we follow the units $c = G = \hbar = 1$.

2. INFLATION IN BIANCHI TYPE-I METRIC

The Bianchi type-I model is an anisotropic generalization of the FRW model with Euclidean spatial geometry. The Bianchi type-I metric can be considered as spatially homogeneous and anisotropic and is given by

$$ds^2 = dt^2 - \mathcal{S}_1^2(t)\, {dx_1}^2 - \mathcal{S}_2^2(t)\, {dx_2}^2 - \mathcal{S}_3^2(t)\, {dx_3}^2, \tag{1}$$

where $\mathcal{S}_1(t)$, $\mathcal{S}_2(t)$ and $\mathcal{S}_3(t)$ are scale factors in three spatial directions. Which are representing the size of the universe in their respective direction. The three scale factors $\mathcal{S}_1(t)$, $\mathcal{S}_2(t)$ and $\mathcal{S}_3(t)$ are determined via Einstein's equations.

In the background metric (1), consider an inflaton, minimally coupled to gravity, satisfy the equation:

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu - m^2)\phi(x,t) = 0, \tag{2}$$

where $\bigtriangledown_\mu$ is the covariant derivative. The Lagrangian density for the inflaton field, ϕ, is given by:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + m^2\phi^2). \tag{3}$$

Since the inflaton is homogeneous, ie.; $\phi(x,t) = \phi(t)$, its classical equation of motion for the metric (1) can be written as

$$\ddot{\phi}(t) + \sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right)\dot{\phi}(t) + m^2\phi(t) = 0. \tag{4}$$

In the present context (4) is the classical equation of motion for the inflaton for the metric (1).

For the metric (1), with temporal component, the classical Einstein equation can be written as:

$$\frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)} + \frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)} + \frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)} = \frac{8\pi T_{00}}{\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)}, \tag{5}$$

where

$$T_{00} = \mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)\left(\frac{\dot{\phi}^2}{2} + m^2\frac{\phi^2(t)}{2}\right), \tag{6}$$

is the energy density of the inflaton. In cosmological context, the classical Einstein equation (5), means that the Hubble constants ($H_i = \frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}$) are determined by the energy density of the dynamically evolving inflaton described by the classical equation of motion.

In order to study the full quantum effects in a cosmological model, both metric and matter fields are to be treated quantum mechanically. Since a consistent quantum theory of gravity is not available, in most of the cosmological models the background metric under consideration is taken as classical form and matter field treat as quantum mechanical. Such approximation of the Einstein equation is know as semiclassical approximation.

In semiclassical theory Einstein equation takes the following form

$$G_{\mu\nu} = 8\pi\langle\hat{T}_{\mu\nu}\rangle, \tag{7}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor and $\langle\hat{T}_{\mu\nu}\rangle$ is the expectation value of the energy-momentum tensor for a matter field in a suitable quantum state under consideration. In (7) the quantum field can be represented by a scalar

field, ϕ, and is governed by the time dependent Schrodinger equation

$$i\frac{\partial}{\partial t}\Psi(\phi,t)=\hat{H}_m(\phi,t)\Psi(\phi,t). \tag{8}$$

Using the canonical quantization procedure, the scalar field can be quantized by defining the momentum conjugate to ϕ, as

$$\pi_\phi=\frac{\partial\mathcal{L}}{\partial\dot{\phi}}. \tag{9}$$

Thus the inflaton in the Bianchi type-I cosmological model, can be described by a time dependent harmonic oscillator with the Hamiltonian

$$H_m=\frac{1}{2\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)}\pi_\phi^2+\frac{m^2\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)}{2}\phi^2(t). \tag{10}$$

The eigenstates of the Hamiltonian are the Fock states which can be constructed by annihilation and creation operators in the following manner.

$$\begin{aligned}\hat{a}(t)&=\phi^*(t)\hat{\pi}_\phi-\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)\dot{\phi}^*(t)\hat{\phi},\\ \hat{a}^\dagger(t)&=\phi(t)\hat{\pi}_\phi-\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)\dot{\phi}(t)\hat{\phi}.\end{aligned} \tag{11}$$

Thus the Fock state of the Hamiltonian is:

$$\hat{a}^\dagger\hat{a}|n,\phi,t\rangle=n|n,\phi,t\rangle. \tag{12}$$

In the present context the semiclassical Einstein equation takes following form

$$\frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)}+\frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)}+\frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)}=\frac{8_\pi\langle\hat{H}\rangle}{\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)}, \tag{13}$$

where H is given by (10).

3. PARTICLE CREATION IN BIANCHI TYPE-I METRIC

Most of the cosmological models are based on a classical behavior of the scalar field. Therefore, it is of interest to study the evolution of the system with the scalar field, which possess the nonclassical features. Recently squeezed states formalism of quantum optics (Shumaker, 1986) is found much useful to deal with quantum issues in cosmology (Albrecht *et al*., 1994; Brandenberger *et al*., 1992; Gasperini and Giovananni, 1993; Grishchuk and Sidorov, 1990; Hu *et al*., 1994; Kim and Page, 1999; Kuo and Ford, 1993; Shumaker, 1986; Suresh, 2001; Suresh and Kuriakose, 1998; Suresh *et al*., 1995). Squeezed states are minimum uncertainty states and are obeying Heisenberg uncertainty principle. A squeezed state is generated by the action of squeezed operator on any coherent state and,

in particular, on the vacuum state. Therefore a squeezed vacuum can be defined (Shumaker, 1986)

$$|\mathcal{Z}\rangle = \mathcal{Z}(r,\varphi)|0\rangle, \tag{14}$$

where the squeezing operator,

$$\mathcal{Z}(r,\varphi) = \exp\left(\frac{r}{2}(e^{-i\varphi}a^2 - e^{i\varphi}a^{\dagger 2})\right). \tag{15}$$

In (15) r is the squeezing parameter which determines the strength of squeezing and φ is the squeezing angle which determines the distribution between the conjugate variables and they take values $0 \le r < \infty$ and $-\pi \le \varphi \le \pi$. a and $a^\dagger$ are respectively known as annihilation and creation operators. When the squeezing operator acts on annihilation and creation operators, lead following results (Shumaker, 1986)

$$\begin{aligned} \mathcal{Z}^\dagger a \mathcal{Z} &= a\cosh r - a^\dagger e^{i\varphi}\sinh r, \\ \mathcal{Z}^\dagger a^\dagger \mathcal{Z} &= a^\dagger \cosh r - a e^{-i\varphi}\sinh r. \end{aligned} \tag{16}$$

In the case of squeezed states, the variance of the quadrate components are not equal but one component of the noise is always squeezed with respect to another.

Next, consider the Hamiltonian of the semiclassical Einstein Eq. (13), whose expectation value can be computed in squeezed vacuum state by replacing the number state $|n,\phi,t\rangle|$ with $|\mathcal{Z}\rangle$. Therefore using (14), (15) and (16) in (13), we obtain the semiclassical Einstein equation as:

$$\begin{aligned} \frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)} + \frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)} + \frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)} &= 8\pi\Bigg[\left(\sinh^2 r + \frac{1}{2}\right)(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) \\ &\quad + \mathrm{Re}\{\cosh r \sinh r\ e^{-i\varphi}(\dot{\phi}^2 + m^2\phi^2)\}\Bigg] \end{aligned} \tag{17}$$

In the above equation, ϕ and ϕ^* satisfy the boundary condition

$$\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)(\dot{\phi}^*(t)\phi(t) - \phi^*(t)\dot{\phi}(t)) = i. \tag{18}$$

To solve the semiclassical Eq. (17), transform the solution in the following form

$$\phi(t) = [\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t)]^{-1/2}\chi(t) \tag{19}$$

Therefore (4) becomes

$$\ddot{\chi}(t) + \left[m^2 + \frac{1}{4}\sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i}{\mathcal{S}_i}\right)^2 - \frac{1}{2}\sum_{i\neq j=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\frac{\dot{\mathcal{S}}_j(t)}{\mathcal{S}_j(t)}\right) - \frac{1}{2}\sum_{i=1}^{3}\frac{\ddot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right]\chi(t) = 0. \tag{20}$$

The inflaton has a solution of the form

$$\chi(t) = \frac{1}{\sqrt{2\gamma}(t)} \exp\left(-i \int \gamma(t)\, dt\right), \tag{21}$$

where,

$$\gamma^2(t) = m^2 + \frac{1}{4}\sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right)^2 - \frac{1}{2}\sum_{i\neq j=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\frac{\dot{\mathcal{S}}_j(t)}{\mathcal{S}_j(t)}\right)$$
$$-\frac{1}{2}\sum_{i=1}^{3}\left(\frac{\ddot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right) + \frac{3}{4}\left(\frac{\dot{\gamma}(t)}{\gamma(t)}\right)^2 - \frac{3}{2}\frac{\ddot{\gamma}(t)}{2\gamma(t)},$$

with the following condition:

$$m^2 > \frac{1}{4}\sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i}{\mathcal{S}_i}\right)^2 - \frac{1}{2}\sum_{i\neq j=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\frac{\dot{\mathcal{S}}_j(t)}{\mathcal{S}_j(t)}\right) - \frac{1}{2}\sum_{i=1}^{3}\frac{\ddot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}. \tag{22}$$

The semiclassical Eq. (17) can be rewritten as follows:

$$\mathcal{S}_1(t)\mathcal{S}_2(t)\mathcal{S}_3(t) = \frac{8\pi}{2\gamma\left(\frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)} + \frac{\dot{\mathcal{S}}_2(t)}{\mathcal{S}_2(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)} + \frac{\dot{\mathcal{S}}_1(t)}{\mathcal{S}_1(t)}\frac{\dot{\mathcal{S}}_3(t)}{\mathcal{S}_3(t)}\right)}\left[\left(\sinh^2 r + \frac{1}{2}\right)\right.$$
$$\times\left[\frac{1}{4}\sum_{i,j=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\frac{\dot{\mathcal{S}}_j(t)}{\mathcal{S}_j(t)}\right) + \frac{3}{4}\sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right)\frac{\dot{\gamma}}{\gamma} + \frac{1}{4}\left(\frac{\dot{\gamma}}{\gamma}\right)^2 + m^2 + \gamma^2\right]$$
$$+ (\cosh r \sinh r \cos(\varphi + 2\gamma t))\left[\frac{1}{4}\sum_{i,j=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\frac{\dot{\mathcal{S}}_j(t)}{\mathcal{S}_j(t)}\right)\right.$$
$$\left.\left. + \frac{3}{4}\sum_{i=1}^{3}\left(\frac{\dot{\mathcal{S}}_i(t)}{\mathcal{S}_i(t)}\right)\frac{\dot{\gamma}}{\gamma} + \frac{1}{4}\left(\frac{\dot{\gamma}}{\gamma}\right)^2 + m^2 - \gamma^2\right]\right]. \tag{23}$$

The above equation can be solved peturbatively. Starting from the approximation ansatzs $\mathcal{S}_{10}(t) = \mathcal{S}_{10}t^{n_1}$, $\mathcal{S}_{20}(t) = \mathcal{S}_{20}t^{n_2}$, $\mathcal{S}_{30}(t) = \mathcal{S}_{30}t^{n_3}$, and $\gamma_0(t) = m$, we obtain the next order perturbative solution for $\mathcal{S}_1$,

$$\mathcal{S}_{11}(t) = \frac{8\pi m}{\mathcal{S}_{20}\mathcal{S}_{30}(n_1n_2 + n_2n_3 + n_1n_3)}\left[\left(\sinh^2 r + \frac{1}{2}\right)\left(1 + \frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2}\right)\right.$$
$$\left. + \cosh r \sinh r\ \cos(\varphi + 2mt)\left(\frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2} - 1\right)\right] t^{2-n_2-n_3}. \tag{24}$$

Similarly, the next order perturbation solution for $\mathcal{S}_2$ and $\mathcal{S}_3$ are obtained as

$$\mathcal{S}_{21}(t) = \frac{8\pi m}{\mathcal{S}_{10}\mathcal{S}_{30}(n_1n_2 + n_2n_3 + n_1n_3)}\left[\left(\sinh^2 r + \frac{1}{2}\right)\left(1 + \frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2}\right)\right.$$
$$\left. + \cosh r \sinh r \cos(\varphi + 2mt)\left(\frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2} - 1\right)\right] t^{2-n_1-n_3}, \quad (25)$$

and

$$\mathcal{S}_{31}(t) = \frac{8\pi m}{\mathcal{S}_{10}\mathcal{S}_{20}(n_1n_2 + n_2n_3 + n_1n_3)}\left[\left(\sinh^2 r + \frac{1}{2}\right)\left(1 + \frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2}\right)\right.$$
$$\left. + \cosh r \sinh r \cos(\varphi + 2mt)\left(\frac{\sum_{i=j=1}^{3} n_i n_j}{8m^2t^2} - 1\right)\right] t^{2-n_1-n_2}. \quad (26)$$

Where $\mathcal{S}_{11}$ means the next order perturbation solution for the scale factor in the x direction and the same holds for $\mathcal{S}_{21}$ and $\mathcal{S}_{31}$; respectively in the y and z directions.

From (25), (26) and (27) it follows that

$$S_{11} \sim t^{2-n_2-n_3}, \qquad S_{21} \sim t^{2-n_1-n_3}, \qquad S_{31} \sim t^{2-n_1-n_2}. \quad (27)$$

The next goal is to examine particle creation in the anisotropically evolving universe described through the Bianchi type-I metric. For this, consider the Fock space which has a one parameter dependence on the cosmological time t. Then the number of particles at a later time t created from the vacuum at the initial time t_0 is given by

$$N_0(t, t_0) = \langle 0, \phi, t_0|\hat{N}(t)|0, \phi, t_0\rangle, \quad (28)$$

where $\hat{N}(t) = a^\dagger a$. Thus using (11), the vacuum expectation value of the right hand side of (29) becomes

$$\langle \hat{N}(t)\rangle = (S_1S_2S_3)^2\dot{\phi}\dot{\phi}^*\langle\hat{\phi}^2\rangle + \phi\phi^*\langle\hat{\pi}\rangle^2 - S_1S_2S_3\phi\dot{\phi}^*\langle\hat{\pi}\hat{\phi}\rangle - S_1S_2S_3\dot{\phi}\phi^*\langle\hat{\phi}\hat{\pi}\rangle. \quad (29)$$

Therefore

$$N_0(t, t_0) = (S_1S_2S_3)^2|\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2. \quad (30)$$

The number of particles created in the vacuum states can be obtained by using the peturbative solution in the limit $mt_0, mt > 1$, in (30), therefore,

$$N_0(t, t_0) = \frac{1}{4_\gamma(t)_\gamma(t_0)}\frac{S_1S_2S_3}{S_{10}S_{20}S_{30}}\left[\frac{1}{4}\sum_{i=j=1}^{3}\left(\frac{\dot{S}_i\dot{S}_j}{S_iS_j} + \frac{\dot{S}_{i0}\dot{S}_{j0}}{S_{i0}S_{j0}}\right) - \frac{1}{2}\sum_{i=j-1}^{3}\frac{\dot{S}_i\dot{S}_{j0}}{\dot{S}_i\dot{S}_{j0}}\right.$$

$$+\frac{1}{2}\sum_{i=1}^{3}\frac{\dot{S}_i}{S_i}\left(\frac{\dot{\gamma}(t)}{\gamma(t)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\right)+\frac{1}{2}\sum_{i=1}^{3}\frac{\dot{S}_{i0}}{S_{i0}}\left(\frac{\dot{\gamma}(t)}{\gamma(t)}-\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\right)$$
$$+\left[\frac{1}{2}\left(\frac{\dot{\gamma}(t)}{\gamma(t)}-\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\right)\right]^2+\gamma(t)^2-\gamma(t_0)^2\Bigg]$$
$$\simeq\left(\frac{n_1+n_2+n_3}{4_m}\right)^2\left(\frac{t-t_0}{tt_0}\right)^2\left(\frac{t}{t_0}\right)^{n_1+n_2+n_3} \tag{31}$$

By a similar procedure, one can compute the particle creation for the quantized inflaton in squeezed vacuum states also. For this, consider the quantized inflaton in the squeezed vacuum state formalism. Then the expectation values of π^2, ϕ^2, $\pi\phi$ and $\phi\pi$ can be computed in squeezed vacuum state by using (11), (14) and (16), and applying the results in (30), the number of particle created in squeezed vacuum in the limit mt_0, $mt > 1$, is obtained as

$$N_{\mathrm{sqv}}(t,t_0)=\frac{1}{4\gamma(t)\gamma(t_0)}\frac{S_1S_2S_3}{S_{10}S_{20}S_{30}}\Bigg[(2\sinh^2 r+1)\Bigg(\frac{1}{4}\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_i(t_0)}{S_i(t_0)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\Bigg)^2$$
$$+\gamma^2(t_0)+\gamma^2(t)+\frac{1}{4}\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_i(t)}{S_i(t)}+\frac{\dot{\gamma}(t)}{\gamma(t)}\Bigg)^2\Bigg)$$
$$+\sinh r\,\cosh r(e^{i(\varphi-2\int\gamma(t)dt)}-e^{-i(\varphi-2\int\gamma(t)\,dt)})$$
$$\times\Bigg(\frac{1}{4}\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_i(t_0)}{S_i(t_0)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\Bigg)^2+\frac{1}{4}\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_i(t)}{S_i(t)}+\frac{\dot{\gamma}(t)}{\gamma(t)}\Bigg)^2$$
$$+\gamma^2(t)-\gamma^2(t_0)\Bigg)-\sinh^2 r\Bigg(\frac{1}{2}\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_i(t)}{S_i(t)}+\frac{\dot{\gamma}(t)}{\gamma(t)}\Bigg)$$
$$\times\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_{i0}(t)}{S_{i0}(t)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\Bigg)-2\gamma(t)\gamma(t_0)\Bigg)-\cosh^2 r\Bigg(\frac{1}{2}\Bigg(\sum_{i=1}^{2}\frac{\dot{S}_i(t)}{S_i(t)}$$
$$+\frac{\dot{\gamma}(t)}{\gamma(t)}\Bigg)\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_{i0}(t)}{S_{i0}(t)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\Bigg)+2\gamma(t)\gamma(t_0)\Bigg)$$
$$-\sinh r\cosh r(e^{i(\varphi-2\int\gamma(t)\,dt))}-e^{-i(\varphi-2\int\gamma(t)\,dt)})$$
$$\Bigg(\frac{1}{2}\sum_{i=1}^{3}\frac{\dot{S}_i(t)}{S_i(t)}+\frac{\dot{\gamma}(t)}{\gamma(t)}\Bigg)\Bigg(\sum_{i=1}^{3}\frac{\dot{S}_{i0}(t)}{S_{i0}(t)}+\frac{\dot{\gamma}(t_0)}{\gamma(t_0)}\Bigg)\Bigg)\Bigg]. \tag{32}$$

Again using the peturbative solution and ansatzs, we get

$$N_{\rm sqv}(t,t_0) \simeq \left(\frac{n_1+n_2+n_3}{4m}\right)^2 \left(\frac{t}{t_0}\right)^{n_1+n_2+n_3} \left(\frac{t-t_0}{tt_0}\right)^2 \times \left[1+2\sinh^2 r+\sinh(2r)\cos(\varphi-2mt)\right]. \simeq N(t,t_0)\left[1+2\sinh^2 r+\sinh(2r)\cos(\varphi-2mt)\right], \quad (33)$$

where $N(t, t_0)$ is given by (31).

4. CONCLUSIONS

In this paper we have examined the behavior of a homogeneous and massive scalar (inflaton) field minimally coupled to the gravity in Bianchi type-I model of the universe, in the frame work of semiclassical theory of gravity. The inflaton is represented in squeezed vacuum state formalism of quantum optics and hence the approximate leading solution to the semiclassical Einstein equation is found. The next order solution for each scale factor in their respective direction show that each scale factor in each direction is dependent on power law of expansion. Further more the solutions show that evolution of scale factors are mutually correlated. When $n_1 = n_2 = n_3 = n$, then the corresponding solution reduces to isotropic model and is consistent with the result obtained by Kim and Page (Kim and Page, 1999). From anisotropic to isotropic transition a damping mechanism is required. One of the efficient damping mechanisms could be due to the particle creation in anisotropic models. We have also examined the nonclassical particle creation in Bianchi type-I cosmological model by representing the inflaton in squeezed vacuum state formalism. The present study can account for the nonclassical particle creation and power law of expansion of the scale factors in Bianchi type-I universe, for a homogeneous and massive scalar field minimally coupled to the gravity, in the frame work of semiclassical theory of gravity.

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